



IB Mathematics

Extended essay guidelines



Nature of the extended essay

The extended essay is an in-depth study of a focused topic chosen from the list of approved Diploma Programme subjects—normally one of the student's six chosen subjects for the IB diploma. It is intended to promote high-level research and writing skills, intellectual discovery and creativity. It provides students with an opportunity to engage in personal research in a topic of their own choice, under the guidance of a supervisor (a teacher in the school). This leads to a major piece of formally presented, structured writing, in which ideas and findings are communicated in a reasoned and coherent manner, appropriate to the subject chosen. It is recommended that completion of the written essay is followed by a short, concluding interview, or viva voce, with the supervisor. The extended essay is assessed against common criteria, interpreted in ways appropriate to each subject.

The extended essay is:

- compulsory for all Diploma Programme students
- externally assessed and, in combination with the grade for theory of knowledge, contributes up to three points to the total score for the IB diploma
- a piece of independent research/investigation on a topic chosen by the student in cooperation with a supervisor in the school
- chosen from the list of approved Diploma Programme subjects, published in the Handbook of procedures for the Diploma Programme
- presented as a formal piece of scholarship containing no more than 4,000 words
- the result of approximately 40 hours of work by the student
- concluded with a short interview, or viva voce, with the supervising teacher (recommended).

In the Diploma Programme, the extended essay is the prime example of a piece of work where the student has the opportunity to show knowledge, understanding and enthusiasm about a topic of his or her choice. In those countries where it is the norm for interviews to be required prior to acceptance for employment or for a place at university, the extended essay has often proved to be a valuable stimulus for discussion.

Aims

The aims of the extended essay are to provide students with the opportunity to:

- pursue independent research on a focused topic
- develop research and communication skills
- develop the skills of creative and critical thinking
- engage in a systematic process of research appropriate to the subject
- experience the excitement of intellectual discovery.

Assessment objectives

In working on the extended essay, students are expected to:

1. plan and pursue a research project with intellectual initiative and insight
2. formulate a precise research question
3. gather and interpret material from sources appropriate to the research question
4. structure a reasoned argument in response to the research question on the basis of the material gathered
5. present their extended essay in a format appropriate to the subject, acknowledging sources in one of the established academic ways
6. use the terminology and language appropriate to the subject with skill and understanding
7. apply analytical and evaluative skills appropriate to the subject, with an understanding of the implications and the context of their research.

Responsibilities of the supervisor

It is required that the supervisor:

- provides the student with advice and guidance in the skills of undertaking research
- encourages and supports the student throughout the research and writing of the extended essay
- discusses the choice of topic with the student and, in particular, helps to formulate a well-focused research question
- ensures that the chosen research question satisfies appropriate legal and ethical standards with regard to health and safety, confidentiality, human rights, animal welfare and environmental issues
- is familiar with the regulations governing the extended essay and the assessment criteria, and gives copies of these to the student
- monitors the progress of the extended essay to offer guidance and to ensure that the essay is the student's own work (this may include presenting a section of the essay for supervisor comment)
- reads and comments on one completed draft only of the extended essay (but does not edit the draft)
- reads the final version to confirm its authenticity
- submits a predicted grade for the student's extended essay to IB
- completes the supervisor's report (if the extended essay cover is not signed by both the student and the supervisor, the essay will not be accepted for assessment and may be returned to the school)

- provides an explanation in the report in cases where the number of hours spent with the student in discussing the extended essay is zero; in particular, it is necessary to describe how it has been possible to guarantee the authenticity of the essay in such circumstances
- writes a report and presents it to the school's Diploma Programme coordinator if malpractice, such as plagiarism, is suspected in the final draft.

Responsibilities of the student

It is required that students:

- choose a topic that fits into one of the subjects on the approved extended essay list
- observe the regulations relating to the extended essay
- meet deadlines
- acknowledge all sources of information and ideas in an approved academic manner.

It is strongly recommended that students:

- start work early
- think very carefully about the research question for their essay
- plan how, when and where they will find material for their essay
- plan a schedule for both researching and writing the essay, including extra time for delays and unforeseen problems
- record sources as their research progresses (rather than trying to reconstruct a list at the end)
- have a clear structure for the essay itself before beginning to write
- check and proofread the final version carefully
- make sure that all basic requirements are met (for example, all students should get full marks for the abstract).

Advice to students from examiners

Recommended: things to do

- read the assessment criteria
- read previous essays to identify strengths and possible pitfalls
- spend time working out the research question (imagine the finished essay)
- work out a structure for the essay.

During the research process, and while writing the essay, students should:

- start work early and stick to deadlines
- maintain a good working relationship with their supervisor
- construct an argument that relates to the research question
- use the library and consult librarians for advice
- record sources as they go along (rather than trying to reconstruct a list at the end)
- choose a new topic and a research question that can be answered if there is a problem with the original topic
- use the appropriate language for the subject
- let their interest and enthusiasm show.

After completing the essay, students should:

- write the abstract
- check and proofread the final version carefully.

Recommended: things to avoid

Examiners' reports also mention these things to be avoided at all costs.

- Students should not work with a research question that is too broad or too vague, too narrow, too difficult or inappropriate. A good research question is one that asks something worth asking and that is answerable within 40 hours/4,000 words. It should be clear what would count as evidence in relation to the question, and it must be possible to acquire such evidence in the course of the investigation. If a student does not know what evidence is needed, or cannot collect such evidence, it will not be possible to answer the research question.

In addition, students should not:

- forget to analyse the research question
- ignore the assessment criteria

- collect material that is irrelevant to the research question
- use the internet uncritically
- plagiarize
- merely describe or report (evidence must be used to support the argument)
- repeat the introduction in the conclusion
- cite sources that are not used.

One further piece of advice is as follows: the more background a student has in the subject, the better the chance he or she has of writing a good extended essay. Choosing to write the extended essay in a subject that is not being studied as part of the Diploma Programme often leads to lower marks.

Researching and writing the extended essay

The research process

When researching the extended essay, students should do the following.

1. Choose the approved Diploma Programme subject for the extended essay.
 - Read the assessment criteria and the relevant subject guidance.
2. Choose a topic.
3. Formulate a well-focused research question.
4. Plan the investigation and writing process.
 - Identify how and where they will gather material.
 - Identify which system of academic referencing they will use, appropriate to the subject of the essay.
 - Set deadlines for themselves that will allow them to meet the school's requirements.
5. Plan a structure (outline headings) for the essay. This may change as the investigation develops but it is useful to have a sense of direction.
6. Undertake some preparatory reading.
 - If students discover that it will not be possible to obtain the evidence needed in the time available, the research question should be changed. This should be done sooner rather than later: students should not lose time waiting and hoping that something will turn up. Students should go back to stage 3, 2 or 1, and choose a new research question that can be answered.

7. Carry out the investigation.

- The material gathered should be assembled in a logical order, linked to the structure of the essay. Only then will students know whether they have enough evidence for each stage of the argument so that they can proceed to the next.
- Students should be prepared for things to go wrong. Sometimes they may discover something later in the investigation that undermines what they thought had been established earlier on. If that happens, the investigation plan needs to be revised.

Writing the extended essay

The structure of the essay is very important. This is what helps students to organize the argument, making best use of the evidence gathered. The required elements of the final work to be submitted are listed here. More details about each element are given in the “Formal presentation of the extended essay” section. Please note that the order in which they are presented here is not necessarily the order in which they should be written.

- Title page
- Abstract
- Contents page
- Introduction
- Body (development/methods/results)
- Conclusion
- References and bibliography
- Appendices

Students should use the chosen system of academic referencing as soon as they start writing. That way, they are less likely to forget to include a citation. It is also easier than trying to add references at a later stage. Most modern word processors are helpful with this.

Some students draft the introduction first. If students do that, they must be prepared to revise it once the essay is complete.

The main task is writing the body of the essay, which should be presented in the form of a reasoned argument. The form of this varies with the subject of the essay but, as the argument develops, it should be clear to the reader what relevant evidence has been discovered, where/how it has been discovered and how it supports the argument. In most subjects, sub-headings within the main body of the essay will help the reader to understand the argument (and will also help the student to keep on track).

Once the main body of the essay is complete, it is possible to finalize the introduction (which tells the reader what to expect) and the conclusion (which says what has been achieved, including notes of any limitations and any questions that have not been resolved).

Any information that is important to the argument should not be included in appendices or footnotes/endnotes.

The examiner is not bound to read notes or appendices, so an essay that is not complete in itself will lose marks.

The remaining stages in writing the essay take time but are not difficult. Students need to check that they have cited sources for all material that is not their own, and that the citations are complete and consistent with the chosen referencing system. The bibliography should list only the sources used in the essay. The whole essay needs to be proofread carefully (computer spelling and grammar checkers are useful but will not do everything). Pages must be numbered and the contents page must be completed. The abstract is normally written last.

Formal presentation of the extended essay

The extended essay should be written in a clear, correct and formal academic style, appropriate to the subject from which the topic is drawn. The use of word processors is encouraged.

The length of the extended essay

The upper limit is 4,000 words for all extended essays. This upper limit includes the introduction, the body, the conclusion and any quotations, but **does not include**:

- the abstract
- acknowledgments
- the contents page
- maps, charts, diagrams, annotated illustrations and tables
- equations, formulas and calculations
- citations/references (whether parenthetical or numbered)
- footnotes or endnotes
- the bibliography
- appendices.

Essays containing more than 4,000 words are subject to penalties and examiners are not required to read material in excess of the word limit.

Title

The title should provide a clear indication of the focus of the essay. It should be precise and not necessarily phrased in the form of a question.

Abstract

An abstract not exceeding 300 words must be included with the essay submitted. It does not serve as an introduction, but presents an overview of the extended essay, and should, therefore, be **written last**.

The inclusion of an abstract is intended to encourage students to examine closely the development of an argument within the extended essay and the pertinence of any conclusions that are reached. It is also designed to allow readers to understand quickly the contents of the extended essay.

The minimum requirements for the abstract are for it to state clearly:

- the research question being investigated
- the scope of the investigation
- the conclusion(s) of the extended essay.

The abstract should be typed or word processed on one side of a sheet of paper, and placed immediately after the title page.

Contents page

A contents page must be provided at the beginning of the extended essay and all pages should be numbered. An index is not required.

Illustrations

Presentation and overall neatness are important, and it is essential that illustrative material, if included, is well set out and used effectively. Graphs, diagrams, tables and maps are effective only if they are clearly labelled and can be interpreted with ease. All such material that is incorporated into the extended essay must be directly related to the text and acknowledged where appropriate. The use of photographs and other images is acceptable only if they are captioned and/or annotated and are used to illustrate a specific point made in the extended essay.

Bibliographies, references and citations

An extended essay must reflect intellectual honesty in research practices and provide the reader with the exact sources of quotations, ideas and points of view through accurate bibliographies and referencing. Producing accurate citations, referencing and a bibliography is a skill that students should be seeking to perfect. Documenting the research in this way is vital: it allows readers to evaluate the evidence for themselves and it shows the student's understanding of the importance of the sources used.

Failure to comply with this requirement will be viewed as plagiarism and will, therefore, be treated as a case of malpractice.

What is a bibliography?

A bibliography is an alphabetical list of every source used to research and write the essay. Sources that are not cited in the body of the essay, but were important in informing the approach taken, should be cited in the introduction or in an acknowledgment. The bibliography should list only those sources cited.

The MLA documentation style should be applied in both the final draft of the essay and in the initial research stages of taking notes. This is good practice, not only for producing a high-quality final product, but also for reducing the opportunities and temptation to plagiarize.

More information about the MLA referencing system can be found here:

<http://owl.english.purdue.edu/owl/resource/747/01/>

What is a reference?

A reference is a way of indicating to the reader, in an orderly form, where information has been obtained. A reference provides all the information needed to find the source material. References must be cited because they acknowledge the sources used, and enable the reader to consult the work and verify the data that has been presented.

References must be given whenever someone else's work is quoted or summarized. References can come from many different sources, including books, magazines, journals, newspapers, emails, internet sites and interviews.

Internet references should include the title of the extract used as well as the website address, the date it was accessed and, if possible, the author. Caution should be exercised with information on websites that do not give references or that cannot be cross-checked against other sources. The more important a particular point is to the essay, the more the quality of its source needs to be evaluated.

Any references to interviews should state the name of the interviewer, the name of the interviewee, the date and the place of the interview.

What is a citation?

A citation is a shorthand method of making a reference in the body of an essay, which is then linked to the full reference at the end of the essay. A citation provides the reader with accurate references so that he or she can locate the source easily. How sources are cited varies with the particular documentation style that has been chosen.

Page numbers should normally be given when referencing printed material: in some styles this will be in the citation, in others in the full reference. Once again, it is important to emphasize that there must be consistency of method when citing sources.

Appendices, footnotes and endnotes

Appendices, footnotes and endnotes are not an essential section of the extended essay and examiners are not required to read them, so care should be taken to include all information of direct relevance to the analysis and argument in the main body of the essay. An essay that attempts to evade the word limit by including important material in notes or appendices risks losing marks under several criteria.

Unless considered essential, complete lists of raw data should not be included in the extended essay.

Students should not constantly refer to material presented in an appendix as this may disrupt the continuity of the essay.

The *viva voce* (concluding interview)

The *viva voce* is a short interview between the student and the supervisor, and is a recommended conclusion to the extended essay process. Students who do not attend the *viva voce* may be disadvantaged.

The *viva voce* serves the following purposes.

- A check on plagiarism and malpractice in general
- An opportunity to reflect on successes and difficulties in the research process
- An opportunity to reflect on what has been learned
- An aid to the supervisor's report

The *viva voce* should last between 10 and 15 minutes. This is included in the recommended amount of time the supervisor should spend with the student. The following are examples of questions that can be asked, which should be adapted to the particular essay and student.

- "I am not clear what you mean on page XXX. You quote Y: could you explain a little more about what this tells us?"

- “On page *** you cite Z. I couldn’t find this reference (for example, website). Could you tell me more about it?”
- “What have been the high and low points of the research and writing processes?”
- “What were the most interesting aspects of the process? Did you discover anything that surprised you?”
- “What have you learned through writing this essay? Is there any advice you would want to pass on to someone just starting out on an extended essay?”
- “Is there anything else that you would particularly like me to mention in my report?”

In conducting the viva voce and writing the report, supervisors should bear in mind the following.

- Examiners want to know that students understand any material (which must be properly referenced) that they have included in their essays. This is particularly important in subjects like mathematics. If the way the material is used in context in the essay does not clearly establish this, the supervisor can check the student’s understanding in the viva voce and report on it.
- Minor slips in citation and referencing may lose the odd mark. If there appear to be major shortcomings, the supervisor should investigate thoroughly. No essay should be authenticated if the supervisor believes it contains plagiarism.
- In assessing criterion K (holistic judgment), examiners will take into account any information given in the report about unusual intellectual inventiveness or persistence in the face of unexpected difficulties.
- The report should not attempt to do the examiner’s job. It should refer to things, largely process-related, that may not be obvious in the essay itself.
- Unless there are particular problems, the viva voce should end positively. Completion of a major piece of work such as the extended essay is something for students to feel good about.

Assessment criteria

All extended essays are externally assessed by examiners appointed by the IB, and are marked on a scale from 0 to 36. This maximum score is made up of the total criterion levels available for each essay. The total score obtained on the scale 0 to 36 is used to determine in which of the following bands the extended essay is placed.

This band, in conjunction with the band for theory of knowledge, determines the number of diploma points awarded for these two requirements. See the following “Award of diploma points” section for further details.

The band descriptors are:

- A** Work of an excellent standard
- B** Work of a good standard
- C** Work of a satisfactory standard
- D** Work of a mediocre standard
- E** Work of an elementary standard.

Award of diploma points

The extended essay contributes to the overall diploma score through the award of points in conjunction with theory of knowledge. A maximum of three points are awarded according to a student’s combined performance in both the extended essay and theory of knowledge.

Both the extended essay and theory of knowledge are measured against published assessment criteria. According to the quality of the work, and based on the application of these assessment criteria, a student’s performance in each of the extended essay and theory of knowledge will fall into one of the five bands described previously.

The total number of points awarded is determined by the combination of the performance levels achieved by the student in both the extended essay and theory of knowledge according to the following matrix.

		Theory of knowledge					
		Excellent A	Good B	Satisfactory C	Mediocre D	Elementary E	Not submitted
Extended essay	Excellent A	3	3	2	2	1 + Failing condition*	N
	Good B	3	2	1	1	Failing condition*	N
	Satisfactory C	2	1	1	0	Failing condition*	N
	Mediocre D	2	1	0	0	Failing condition*	N
	Elementary E	1 + Failing condition*	Failing condition*	Failing condition*	Failing condition*	Failing condition*	N
	Not submitted	N	N	N	N	N	N

A student who, for example, writes a good extended essay and whose performance in theory of knowledge is judged to be satisfactory will be awarded 1 point, while a student who writes a mediocre extended essay and whose performance in theory of knowledge is judged to be excellent will be awarded 2 points.

A student who fails to submit an extended essay will be awarded N for the extended essay, will score no points, and will not be awarded a diploma.

Performance in both the extended essay and theory of knowledge of an elementary standard is a failing condition for the award of the diploma.

* From 2010 onwards, 28 points overall will be required to be eligible for the diploma if a student attains an “E” grade in either the extended essay or theory of knowledge. As previously, a grade “A” in one of the requirements earns an extra point even if the other is a grade “E”. Attaining a grade “E” in both the extended essay and theory of knowledge continues to represent an automatic failure.

Criterion A: research question

This criterion assesses the extent to which the purpose of the essay is specified. In many subjects, the aim of the essay will normally be expressed as a question and, therefore, this criterion is called the “research question”. However, certain disciplines may permit or encourage different ways of formulating the research task.

Achievement level	Descriptor
0	The research question is not stated in either the introduction or on the title page or does not lend itself to a systematic investigation in an extended essay in the subject in which it is registered .
1	The research question is stated in either the introduction or on the title page but is not clearly expressed or is too broad in scope to be treated effectively within the word limit .
2	The research question is clearly stated in either the introduction or on the title page and is sharply focused, making effective treatment possible within the word limit .

Criterion B: introduction

This criterion assesses the extent to which the introduction makes clear how the research question relates to existing knowledge on the topic and explains how the topic chosen is significant and worthy of investigation.

Achievement level	Descriptor
0	Little or no attempt is made to set the research question into context. There is little or no attempt to explain the significance of the topic .
1	Some attempt is made to set the research question into context. There is some attempt to explain the significance of the topic and why it is worthy of investigation .
2	The context of the research question is clearly demonstrated. The introduction clearly explains the significance of the topic and why it is worthy of investigation .

Criterion C: investigation

This criterion assesses the extent to which the investigation is planned and an appropriate range of sources has been consulted, or data has been gathered, that is relevant to the research question. Where the research question does not lend itself to a systematic investigation in the subject in which the essay is registered, the maximum level that can be awarded for this criterion is 2.

Achievement level	Descriptor
0	There is little or no evidence that sources have been consulted or data gathered, and little or no evidence of planning in the investigation.
1	A range of inappropriate sources has been consulted, or inappropriate data has been gathered, and there is little evidence that the investigation has been planned.
2	A limited range of appropriate sources has been consulted, or data has been gathered, and some relevant material has been selected. There is evidence of some planning in the investigation.
3	A sufficient range of appropriate sources has been consulted, or data has been gathered, and relevant material has been selected. The investigation has been satisfactorily planned.
4	An imaginative range of appropriate sources has been consulted, or data has been gathered, and relevant material has been carefully selected. The investigation has been well planned.

Criterion D: knowledge and understanding of the topic studied

Where the research question does not lend itself to a systematic investigation in the subject in which the essay is registered, the maximum level that can be awarded for this criterion is 2. "Academic context", as used in this guide, can be defined as the current state of the field of study under investigation. However, this is to be understood in relation to what can reasonably be expected of a pre-university student. For example, to obtain a level 4, it would be sufficient to relate the investigation to the principal lines of inquiry in the relevant field; detailed, comprehensive knowledge is not required.

Achievement level	Descriptor
0	The essay demonstrates no real knowledge or understanding of the topic studied.
1	The essay demonstrates some knowledge but little understanding of the topic studied. The essay shows little awareness of an academic context for the investigation.
2	The essay demonstrates an adequate knowledge and some understanding of the topic studied. The essay shows some awareness of an academic context for the investigation.
3	The essay demonstrates a good knowledge and understanding of the topic studied. Where appropriate, the essay successfully outlines an academic context for the investigation.
4	The essay demonstrates a very good knowledge and understanding of the topic studied. Where appropriate, the essay clearly and precisely locates the investigation in an academic context.

Criterion E: reasoned argument

This criterion assesses the extent to which the essay uses the material collected to present ideas in a logical and coherent manner, and develops a reasoned argument in relation to the research question. Where the research question does not lend itself to a systematic investigation in the subject in which the essay is registered, the maximum level that can be awarded for this criterion is 2.

Achievement level	Descriptor
0	There is no attempt to develop a reasoned argument in relation to the research question.
1	There is a limited or superficial attempt to present ideas in a logical and coherent manner, and to develop a reasoned argument in relation to the research question.
2	There is some attempt to present ideas in a logical and coherent manner, and to develop a reasoned argument in relation to the research question, but this is only partially successful.
3	Ideas are presented in a logical and coherent manner, and a reasoned argument is developed in relation to the research question, but with some weaknesses.
4	Ideas are presented clearly and in a logical and coherent manner. The essay succeeds in developing a reasoned and convincing argument in relation to the research question.

Criterion F: application of analytical and evaluative skills appropriate to the subject

Achievement level	Descriptor
0	The essay shows no application of appropriate analytical and evaluative skills.
1	The essay shows little application of appropriate analytical and evaluative skills.
2	The essay shows some application of appropriate analytical and evaluative skills, which may be only partially effective.
3	The essay shows sound application of appropriate analytical and evaluative skills.
4	The essay shows effective and sophisticated application of appropriate analytical and evaluative skills.

Criterion G: use of language appropriate to the subject

Achievement level	Descriptor
0	The language used is inaccurate and unclear. There is no effective use of terminology appropriate to the subject .
1	The language used sometimes communicates clearly but does not do so consistently. The use of terminology appropriate to the subject is only partly accurate .
2	The language used for the most part communicates clearly. The use of terminology appropriate to the subject is usually accurate .
3	The language used communicates clearly. The use of terminology appropriate to the subject is accurate, although there may be occasional lapses .
4	The language used communicates clearly and precisely. Terminology appropriate to the subject is used accurately, with skill and understanding .

Criterion H: conclusion

This criterion assesses the extent to which the essay incorporates a conclusion that is relevant to the research question and is consistent with the evidence presented in the essay.

Achievement level	Descriptor
0	Little or no attempt is made to provide a conclusion that is relevant to the research question.
1	A conclusion is attempted that is relevant to the research question but may not be entirely consistent with the evidence presented in the essay .
2	An effective conclusion is clearly stated; it is relevant to the research question and consistent with the evidence presented in the essay. It should include unresolved questions where appropriate to the subject concerned .

Criterion I: formal presentation

This criterion assesses the extent to which the layout, organization, appearance and formal elements of the essay consistently follow a standard format. The formal elements are: title page, table of contents, page numbers, illustrative material, quotations, documentation (including references, citations and bibliography) and appendices (if used).

Achievement level	Descriptor
0	The formal presentation is unacceptable, or the essay exceeds 4,000 words.
1	The formal presentation is poor.
2	The formal presentation is satisfactory.
3	The formal presentation is good.
4	The formal presentation is excellent.

Criterion J: abstract

The requirements for the abstract are for it to state clearly the research question that was investigated, how the investigation was undertaken and the conclusion(s) of the essay.

Achievement level	Descriptor
0	The abstract exceeds 300 words or one or more of the required elements of an abstract (listed above) is missing.
1	The abstract contains the elements listed above but they are not all clearly stated.
2	The abstract clearly states all the elements listed above.

Criterion K: holistic judgment

The purpose of this criterion is to assess the qualities that distinguish an essay from the average, such as intellectual initiative, depth of understanding and insight. While these qualities will be clearly present in the best work, less successful essays may also show some evidence of them and should be rewarded under this criterion.

Achievement level	Descriptor
0	The essay shows no evidence of such qualities.
1	The essay shows little evidence of such qualities.
2	The essay shows some evidence of such qualities.
3	The essay shows clear evidence of such qualities.
4	The essay shows considerable evidence of such qualities.

Academic honesty

The student is ultimately responsible for ensuring that his or her extended essay is authentic, with the work or ideas of others fully and correctly acknowledged. Additionally, it is the responsibility of a supervisor to confirm that, for each student he or she has supervised, to the best of his or her knowledge, the version of the extended essay submitted for assessment is the authentic work of the student.

Both plagiarism and collusion are forms of malpractice that incur a penalty. The same piece of work, or two versions of the same work, cannot be submitted to meet the requirements of both the extended essay and another assessment component of a subject contributing to the diploma or an additional certificate.

The extended essay in mathematics

Overview

An extended essay in mathematics provides students with an opportunity to demonstrate an appreciation of any aspect of the subject, whether it is:

- the applicability of mathematics to solve both real and abstract problems
- the beauty of mathematics as in, for instance, geometry or fractal theory
- the elegance of mathematics in the proving of theorems as in, for example, number theory
- the origin and subsequent development of a branch of mathematics over a period of time, measured in tens, hundreds or thousands of years
- the links between different branches of mathematics and the powerful structures that enable many seemingly different problems to be solved by a single theory
- the way that a branch of mathematics has been born, or has flourished, as a result of technology.

These are just some of the many different ways that mathematics can be enjoyable or useful, or, as in many cases, both.

Choice of topic

The extended essay may be written on any topic that has a mathematical focus and it need not be confined to the theory of mathematics itself.

Students may choose mathematical topics from fields such as engineering, the sciences or the social sciences, as well as from mathematics itself. Statistical analyses of experimental results taken from other subject areas are also acceptable, provided that they focus on the modelling process and discuss the limitations of the results; such essays should not include extensive non-mathematical detail. A topic selected from the history of mathematics may also be appropriate, provided that a clear line of mathematical development is demonstrated. Concentration on the lives of, or personal rivalries between, mathematicians would be irrelevant and would not score highly on the assessment criteria.

It should be noted that the assessment criteria give credit for the nature of the investigation and for the extent that reasoned arguments are applied to an appropriate research question. Students should avoid choosing a topic that gives rise to a trivial research question or one that is not sufficiently focused to allow appropriate treatment within a suitably sized essay. Students will normally be expected either to extend their knowledge beyond that encountered in the Diploma Programme mathematics course they are studying, or to apply techniques used in their mathematics course to modelling in an appropriately chosen topic. However, it is very important to remember that it is an essay that is being written, not a research paper for a journal of advanced mathematics, and no result, however impressive, should be quoted without evidence of the student's real understanding of it.

The following examples of titles for mathematics extended essays are intended as guidance only. The pairings illustrate that focused topics (indicated by the first title) should be encouraged rather than broad topics (indicated by the second title).

- “Prime numbers in cryptography” **is better than** “Prime numbers”.
- “The Hausdorff dimension of fractal sets” **is better than** “Fractals”.
- “Continued fractions in birth–death processes” **is better than** “Continued fractions”.
- “The proof of the law of quadratic reciprocity” **is better than** “CF Gauss: the mathematician”.
- “Using graph theory to minimize cost” **is better than** “Graph theory”.

The next examples illustrate ways in which mathematics topics can be refined and successfully approached as extended essays.

Topic	The geometry of navigation
Research question	What was the role of mathematics, and geometry in particular, in navigation when we relied on the stars? Does it still play a part now we have man-made satellites?
Approach	Using one of the two geometric representations of the earth (spherical or ellipsoidal), describe how maps and charts were produced to assist navigators in the past.
Topic	Square–triangular numbers and Pell’s equation
Research question	How many square numbers are also triangular numbers, where are they, and what other problems lead to Pell’s equation?
Approach	A description of square and triangular numbers, and how the locations of numbers that are both are solutions of Pell’s equation. Some other problems, perhaps in number theory and geometry, that lead to the equation could be described, with a brief history of the equation included.
Topic	The exponential function and the measurement of age and growth
Research question	How does the exponential function, and its calculus, inform areas of science such as nuclear physics, geology, anthropology or demography?
Approach	Use one of the settings where exponential growth applies, perhaps modelling the world’s population, to describe the phenomena. Show how it is applicable in mathematical models of other real situations.

Topic	Approximation of irrational numbers by rational numbers
Research question	How well can π , e , $\sqrt{2}$ and other irrationals be approximated by rational numbers?
Approach	Use the decimal representation of irrational numbers as a starting point to introduce approximation by rationals. Show how a continued fraction expansion of an irrational can also provide rational approximation, and discuss error bounds and orders of approximation.
Topic	Archimedes' calculations of areas
Research question	What is the legacy of Archimedes' calculations of circular and parabolic areas in today's methods of integration?
Approach	Describe how Archimedes determined the area of a circle by using inscribed polygons, leading also to his measurement of π . Continue with a description of his method of discovery for calculating the area of a parabola.

Treatment of the topic

Whatever the title of the extended essay, students must apply good mathematical practice that is relevant to the chosen topic. Data must be analysed using appropriate techniques; arguments correctly reasoned; situations modelled using correct methodology; problems clearly stated and techniques at the correct level of sophistication applied to their solution. There must be sufficient explanation and commentary throughout the extended essay to ensure that the reader does not lose sight of the purpose of the essay in a mass of mathematical symbols, formulae and analysis.

The unique disciplines of mathematics must be respected throughout. Relevant graphs and diagrams are often important and should be incorporated in the body of the essay, not relegated to an appendix. However, lengthy printouts, tables of results and computer programs should not be allowed to interrupt the development of the essay, and should appear separately as footnotes or in an appendix. Proofs of key results may be included but proofs of standard results should be either omitted or, if they illustrate an important point, included in an appendix. Frequent reference to the assessment criteria by both the supervisor and the student will help keep a sharper focus on the project.

Interpreting the assessment criteria

Criterion A: research question

Although the aim of the essay can best be defined in the form of a question, it may also be presented as a statement or proposition for discussion.

Criterion B: introduction

The introduction should relate the research question to existing subject knowledge: the student's personal experience or particular opinion is rarely relevant here.

Criterion C: investigation

The number of sources that require consultation will be dependent on the research question that the essay addresses. It must be sufficient, but not unnecessarily long, and the various items must, after being consulted, contribute to the essay in a meaningful way.

Criterion D: knowledge and understanding of the topic studied

Clear evidence of understanding of the chosen content of the essay is more important than any attempt to exhibit an unnecessarily wide mathematical knowledge. The level of knowledge displayed should reflect the mathematics acquired in the classroom and any additional information that the research for the essay has provided.

Criterion E: reasoned argument

In a mathematics extended essay, the terms “reasoning” and “argument” can apply to the essay overall and, with a somewhat different interpretation, to one or more sub-sections of the essay, such as in the proof of a theorem. The mark awarded should reflect both of these aspects.

Criterion F: application of analytical and evaluative skills appropriate to the subject

The words “appropriate to the subject” are paramount here, given the breadth of mathematics today. Essays based on mathematical proof require skills in deductive reasoning and structuring; essays of an investigative nature, answering a research question or hypothesis, involve interpretative skills; while mathematical modelling requires an ability to formulate correctly a problem in mathematical terms.

Criterion G: use of language appropriate to the subject

The purpose of language in mathematics is, as in other subjects, to communicate. In an extended essay, it should communicate clearly, not just to an examiner who is likely to be more mathematically sophisticated than the student, but also to an interested reader with a similar mathematical competence to the student. Mathematical “name-dropping” should be avoided.

Criterion H: conclusion

“Consistent” is the key word here: the conclusion should develop out of the argument and not introduce new or extraneous matter. It should not repeat the material of the introduction; rather, it should present a new synthesis in light of the discussion.

Criterion I: formal presentation

This criterion relates to the extent to which the essay conforms to academic standards about the way in which research papers should be presented. The presentation of essays that omit a bibliography or that do not give references for quotations is deemed unacceptable (level 0). Essays that omit one of the required elements—title page, table of contents, page numbers—are deemed no better than satisfactory (maximum level 2), while essays that omit two of them are deemed poor at best (maximum level 1).

Word count is rarely an important factor in a good mathematics extended essay. Since equations and formulae (indicating the student’s mathematical reasoning) are not included in the word count, a substantial essay can be produced that contains comparatively few words. Concise, elegant mathematics supported by graphs, diagrams and important proofs that do not interrupt the development of the essay are encouraged. However, an essay that is excessive in length will be penalized, especially if this is because of unnecessary content. There is no mandatory minimum length for an essay in mathematics, and credit will be given for organizing the content in an efficient readable style, rather than for a page

or word count. Mastery of appropriate concepts, and an ability to present these in an effective way using mathematical means, should be the aim.

Criterion J: abstract

The abstract is judged on the clarity with which it presents an overview of the research and the essay, not on the quality of the research question itself, nor on the quality of the argument or the conclusions.

Criterion K: holistic judgment

Qualities that are rewarded under this criterion include intellectual initiative, insight and depth of understanding, and creativity. Such qualities can be displayed in different ways, depending on the type of mathematics extended essay.

The information in this document is taken from:

The International Baccalaureate Organization ® *Extended Essay Guide (First examinations 2013)*